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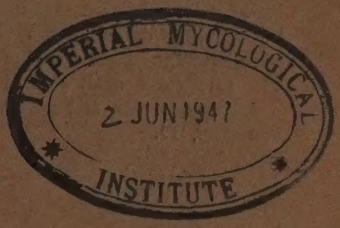

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MATHEMATICS

ON GENERALIZED BERWALD SPACES

By V. WAGNER

(Communicated by S. L. Soboleff, Member of the Academy, 3. IX. 1942)

A Finslerian is said to be a generalized Berwald space if there is possible to introduce a generalized Cartan connexion (1), with the torsion affinor $S^{\cdot\cdot\alpha}_{\gamma\beta} = {}^*\Gamma^{\alpha}_{[\gamma\beta]}$ different from zero, in such a way that the coefficients of connexion ${}^*\Gamma^{\alpha}_{\gamma\beta}$ would be functions of point alone and do not depend on the direction of linear element. If the latter condition is satisfied under $S^{\cdot\cdot\alpha}_{\gamma\beta} = 0$, then the space is said to be Berwald space (2). In the present note we obtain an invariant characteristic of a two-dimensional generalized Berwald space. Let $L(\xi^\alpha, d\xi^\alpha)$ be the metric function of a two-dimensional Finslerian space F_2, $a_{\alpha\beta} = \frac{1}{2}\frac{\partial^2 L^2}{\partial x^\alpha \partial x^\beta}$ be its metric tensor and $\varepsilon_{\alpha\beta}$ its discriminant affinor. The equation

$$L(\xi^\alpha, x^\alpha) = 1 \tag{1}$$

determines a curve in the local E_2 (two-dimensional affine Euclidean space) corresponding to the point ξ^α of F_2, which is known as the indicatrix of F_2. The vector $l^\alpha = \frac{x^\alpha}{L(\xi^\lambda, x^\lambda)}$ is the radius vector of the indicatrix and the vector $t^\alpha = \varepsilon^{\alpha}_{\beta} l^\beta$ is the tangent vector of the indicatrix.

Denoting by ϑ the central affine length of the arc of the indicatrix (3), we have

$$\frac{\partial l^\alpha}{\partial \vartheta} = t^\alpha. \tag{2}$$

If Φ is a certain function of a linear element in F_2, then

$$\frac{\partial \Phi}{\partial \vartheta} = L t^\alpha \frac{\partial \Phi}{\partial x^\alpha}. \tag{3}$$

The main scalar of F_2

$$A = \frac{1}{2}\frac{\partial}{\partial \vartheta} \ln \mathfrak{A}, \qquad \mathfrak{A} = \mathrm{Det}\,|a_{\alpha\beta}| \tag{4}$$

can be interpreted as the central affine curvature of the indicatrix. Denoting $\varepsilon = a_{\alpha\beta} t^\alpha t^\beta$, where $\varepsilon = \pm 1$, we have

$$\frac{\partial t^\alpha}{\partial \vartheta} = -\varepsilon l^\alpha - A t^\alpha. \tag{5}$$

Analogously to (4) and (5) we have

$$\frac{\partial l_\alpha}{\partial \vartheta} = t_\alpha; \qquad \frac{\partial t_\alpha}{\partial \vartheta} = -\varepsilon l_\alpha + A t_\alpha. \tag{6}$$

Let us denote by ${}^*\Gamma^\alpha_{\gamma\beta}$ the coefficients of generalized Cartan connexion in F_2 whose torsion affinor we represent as follows:

$$S^{\cdot\cdot\alpha}_{\gamma\beta} = S_{[\gamma}\delta^\alpha_{\beta]}. \tag{7}$$

We have

$$\begin{aligned} {}^*\Gamma^\alpha_{\gamma\beta} = \left\{{\alpha \atop \gamma\beta}\right\} + S_0\,(At_\gamma t_\beta t^\alpha + \varepsilon t_\gamma t_\beta l^\alpha - \varepsilon t_\gamma l_\beta t^\alpha) + \\ + S_t\,(\varepsilon A^2 t_\gamma t_\beta t_\alpha + At_\gamma t_\beta l^\alpha + \varepsilon l_\gamma l_\beta t^\alpha - \varepsilon l_\gamma t_\beta l^\alpha - 2Al_{(\gamma}t_{\beta)}t^\alpha) \end{aligned} \tag{8}$$

where

$$\left.\begin{aligned} \left\{{\alpha \atop \gamma\beta}\right\} = \frac{1}{2}\,a^{\alpha\omega}\,(\tilde\partial_\gamma a_{\beta\omega} + \tilde\partial_\beta a_{\gamma\omega} - \tilde\partial_\omega a_{\gamma\beta}); \\ S_0 = l^\alpha S_\alpha; \quad S_t = t^\alpha S_\alpha, \end{aligned}\right\} \tag{9}$$

$\tilde\partial_\gamma$ being differential operator defined as follows

$$\left.\begin{aligned} \tilde\partial_\gamma = \frac{\partial}{\partial\xi^\gamma} - \frac{\partial G^\lambda}{\partial x^\gamma}\frac{\partial}{\partial x^\lambda}; \\ G^\lambda = \frac{1}{4}\,a^{\lambda\mu}\left(x^\nu \frac{\partial^2 L^2}{\partial x^\mu\,\partial\xi^\nu} - \frac{\partial L^2}{\partial\xi^\mu}\right). \end{aligned}\right\} \tag{10}$$

It may be shown

$$\frac{\partial}{\partial\vartheta}\left\{{\alpha \atop \gamma\beta}\right\} = (\nabla_t A - A\nabla_0 A)\,t_\gamma t_\beta t^\alpha - \varepsilon\nabla_0 A t_\gamma t_\beta l^\alpha + 2\varepsilon\nabla_0 A l_{(\gamma}t_{\beta)}t^\alpha \tag{11}$$

where

$$\left.\begin{aligned} \nabla_0 A = l^\alpha\nabla_\alpha A; \\ \nabla_t A = t^\alpha\nabla_\alpha A; \\ \nabla_\alpha A = \tilde\partial_\alpha A. \end{aligned}\right\} \tag{12}$$

If F_2 is a generalized Berwald space, then we can choose the torsion vector S_γ in such a way that ${}^*\Gamma^\alpha_{\gamma\beta}$ will be functions of ξ^α alone

$$\frac{\partial\,{}^*\Gamma^\alpha_{\gamma\beta}}{\partial\vartheta} = 0. \tag{13}$$

It follows that S_γ must be also functions of ξ^α alone and therefore, according to (2) and (5),

$$\frac{\partial S_0}{\partial\vartheta} = S_t; \qquad \frac{\partial S_t}{\partial\vartheta} = -\varepsilon S_0 - AS_t. \tag{14}$$

By means of (8), (11), (14), (2), (5) and (6) we obtain that the equation (13) is equivalent to the equations

$$\left.\begin{aligned} \frac{\partial A}{\partial\vartheta}\,S_0 + 2\varepsilon A\frac{\partial A}{\partial\vartheta}\,S_t = A\nabla_0 A - \nabla_t A; \\ \frac{\partial A}{\partial\vartheta}\,S_t = \varepsilon\nabla_0 A. \end{aligned}\right\} \tag{15}$$

Supposing $\dfrac{\partial A}{\partial\vartheta} \neq 0$, we obtain

$$S_0 = -\frac{A\nabla_0 A + \nabla_t A}{\dfrac{\partial A}{\partial\vartheta}}; \qquad S_t = \varepsilon\frac{\nabla_0 A}{\dfrac{\partial A}{\partial\vartheta}}. \tag{16}$$

Substituting these expressions in the equations (14) and using the identity

$$\frac{\partial}{\partial\vartheta}\nabla_\beta\Phi - \nabla_\beta\frac{\partial\Phi}{\partial\vartheta} = -\varepsilon\nabla_0 A t_\beta\frac{\partial\Phi}{\partial\vartheta}, \tag{17}$$

we obtain the following equation:

$$\frac{\partial A}{\partial \theta}\nabla_\beta \frac{\partial A}{\partial \theta} - \frac{\partial^2 A}{\partial \theta^2}\nabla_\beta A = 0 \tag{18}$$

which means that $\frac{\partial A}{\partial \theta}$ is a function of A:

$$\frac{\partial A}{\partial \theta} = f(A). \tag{19}$$

If $\frac{\partial A}{\partial \theta} = 0$, then from the equations (15) we obtain $A = \text{const}$, i. e. F_2 is a Berwald space. In this case S_γ can be chosen as arbitrary functions of ξ^α.

Thus, we have the theorem.

A necessary and sufficient condition that $F_2\left(\frac{\partial A}{\partial \theta} \neq 0\right)$ *be a generalized Berwald space is that* $\frac{\partial A}{\partial \theta}$ *be a function of* A. *If* $\frac{\partial A}{\partial \theta} = 0$, *then* A *must be constant.*

Considering a generalized Berwald space as L_2 (X_2 with a linear connexion) whose coefficients of connexion are equal to ${}^*\Gamma^\alpha_{\gamma\beta}$, we can prove that if $\frac{\partial A}{\partial \theta} \neq 0$, then the group of holonomicity of this L_2 consists of translations only and therefore L_2 is of zero curvature.

F_2, determined by the cubic differential form

$$L(\xi^\alpha, d\xi^\alpha) = \sqrt[3]{a_{\alpha\beta\gamma}\, d\xi^\alpha\, d\xi^\beta\, d\xi^\gamma}, \tag{20}$$

gives an example of generalized Berwald space([4]). In this case the condition (19) has the form

$$\frac{\partial A}{\partial \theta} = -3A^2 - \frac{3}{2}\varepsilon. \tag{21}$$

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MATHEMATICS

ON CERTAIN FIELDS OF GEODESIC PENCILS

By J. L. SHAPIRO

(Communicated by A. N. Kolmogoroff, Member of the Academy, 29. XI. 1942)

1. In a space of n independent variables $x^1, \ldots, x^n$ converted into a metric manifold by the tensor g_{ij} there is given a symmetrical tensor a_{ij} $(i, j = 1, \ldots, n)$. In every point of the space the equation

$$a_{ij}\lambda^i\lambda^j = 0 \tag{1}$$

defines a pencil of directions for which the quantities λ^i are components of a vector, characterizing one of these directions. The field formed by the pencils of directions, defined by the tensor a_{ij}, is said to be geodesic, if all geodesic lines whose direction coincides at a certain point with the direction defined by (1) preserve this property throughout their extension (i. e. the directions of these geodesics continue to coincide with the directions of the field). It can be proved that the necessary and sufficient conditions for this are set by the equations (2)

$$a_{ij,\,k} + a_{kj,\,i} + a_{ik,\,j} = N_k a_{ij} + N_i a_{kj} + N_j a_{ki} \tag{2}$$

where $a_{ij,\,k}$ denote the covariant derivative of the tensor a_{ij}, and N_i are components of an arbitrary vector.

2. Let φ be a given function of the coordinates $\left(\text{such that } g^{ij}\frac{\partial\varphi}{\partial x^i}\frac{\partial\varphi}{\partial x^j} > 1\right)$, and consider the directions for which

$$ds^2 = g_{ij}\,dx^i\,dx^j = d\varphi^2$$

or

$$(g_{ij} - \varphi_{,i}\,\varphi_{,j})\,dx^i\,dx^j = 0, \tag{3}$$

where $\varphi_{,i}$ stands for $\frac{\partial\varphi}{\partial x^i}$.

As a result, we obtain a field of direction pencils defined by the tensor

$$a_{ij} = g_{ij} - \varphi_{,i}\,\varphi_{,j}. \tag{4}$$

It is easily seen that the directions of the pencil (3) form an angle α, which is constant for a given point of the space, and for which

$$\cos\alpha = \frac{1}{\sqrt{g^{ij}\varphi_{,i}\varphi_{,j}}}.$$

The field of directions defined by the tensor a_{ij} is a natural generalization of two-dimensional networks of equal paths [2], since the length of the arc, measured along a line whose directions coincide all along with the directions of the field, depends only upon the start point and the end point of the measuring (the obvious rule of signs is supposed to be fulfilled).

Applying to the tensor a_{ij} the equations (2), one may prove the following proposition.

The necessary and sufficient condition that the field of pencils defined by the tensor $a_{ij}=g_{ij}-\varphi_{,i}\varphi_{,j}$ be geodesic, consists in the equalities

$$\varphi_{,ij}=T(g_{ij}-\varphi_{,i}\varphi_{,j}), \tag{5}$$

where T, if the condition $g^{ij}\varphi_{,i}\varphi_{,j}\neq 1$ is fulfilled, depends solely upon φ.

The last system of equations characterizes a stratifiable space of order 1 (4). The squared length of its linear element, in the special system of coordinates, may be written in the form

$$ds^2=l(d\varphi)^2+l^{2\theta}\Omega_{pq}\,dx^p\,dx^q,\quad i,j=1,\ldots,n-1,$$

where θ and l depend upon φ, and Ω_{pq} depends upon the coordinates $x^1,\ldots,x^{n-1}$.

Conversely, with this metric, there exists a family of geodesic pencils depending upon a single parameter.

3. Stratifiable spaces of order ω (4), possessing a metric

$$ds^2=g_{\alpha\beta}\,d\varphi^\alpha\,d\varphi^\beta+l^{2T}\Omega_{pq}\,dx^p\,dx^q \tag{6}$$

where $g_{\alpha\beta}$ and T depend only upon φ^ν, and Ω_{pq} depends upon x^s *, are also characterized by the existence of a certain field of geodesic pencils. This is formulated in the following proposition.

The existence of the tensor a_{ij} defines a stratifiable space of order ω, if the following four conditions are fulfilled:

I. The squared length of the linear element dx^i belonging to an infinitesimal pencil $a_{ij}dx^i dx^j=0$ is expressed by not degenerating quadratic form of ω ($\omega<n$) independent functions φ^ν, i. e. $ds^2=G_{\alpha\beta}d\varphi^\alpha d\varphi^\beta$, where: $G_{\alpha\beta}=G_{\alpha\beta}(\varphi^\nu)$ and $|G_{\alpha\beta}|\neq 0$.

This is equivalent to the equality

$$(g_{ij}-G_{\alpha\beta}\varphi^\alpha_{,i}\varphi^\beta_{,j})\,dx^i\,dx^j=0,$$

from which it may be concluded that

$$a_{ij}=g_{ij}-G_{\alpha\beta}\varphi^\alpha_{,i}\varphi^\beta_{,j}. \tag{7}$$

II. The characteristic equation for the tensor a_{ij}

$$|a_{ij}-\rho g_{ij}|=0$$

admits a root ρ of multiplicity ω, which is different from zero and unity.

III. The gradients $\varphi^c_{,i}$ form a holonome system of vectors, which can be written thus:

$$\varphi^a_{,ij}\varphi^{cj}-\varphi^c_{,ij}\varphi^{aj}=B^{ac}_\nu\varphi^\nu_{,i}. \tag{8}$$

These equations represent, in an invariant form, the integrability conditions written by using Poisson's «parantheses».

IV. Tensor a_{ij} defines the geodesic field of pencils, or, in other words, (2) is fulfilled. Conversely, a metric of the form (6) defines, up to the constant, the tensor a_{ij} satisfying the four enumerated conditions.

Introducing the notation $g^{ij}\varphi^\alpha_{,i}\varphi^\beta_{,j}=\Delta^{\alpha\beta}$ and denoting by $\bar{\Delta}_{\alpha\beta}$ the matrix inverse to that denoted by $\Delta^{\alpha\beta}$, it may be shown that the second condition is equivalent to

$$G_{\alpha\beta}=(1-\rho)\,\bar{\Delta}_{\alpha\beta}.$$

Note that the characteristic equation for the tensor a_{ij} from the equality (6) admits the root 1 of multiplicity $n-\omega$.

In order to conserve a similar characteristic for stratifiable spaces possessing an indetermined metric, it will suffice to replace the condition II by the two following ones:

a) the rank of the matrix $\Delta^{\alpha\beta}$ is ω;

b) $G_{\alpha\beta}=(1-\rho)\,\bar{\Delta}_{\alpha\beta}$.

---

* The greek indices are assumed here to run over $1,\ldots,\omega$; the indices running over the values $\omega+1\ldots,n$ we denote by p, q, r and s, while the remaining letters of the latin alphabet are reserved for indices running over all the values $1,\ldots n$.

Here ρ is an arbitrary function of the coordinates, not reducible to zero or unity.

From the four conditions stated above it follows that values ρ, $\Delta^{\alpha\beta}$ depend solely upon φ^ν, and, besides, we arrive at the following expression for the second covariant derivatives of these functions:

$$\varphi^\alpha_{,ij} = T^\alpha g_{ij} - \Pi^\alpha_{\varkappa\mu}\varphi^\varkappa_{,i}\varphi^\mu_{,j}, \tag{9}$$

where $T^\alpha = -\frac{1}{2}\frac{1}{\rho(1-\rho)}\Delta^{\alpha\beta}\frac{\partial\rho}{\partial\varphi^\beta}$, and $\Pi^\alpha_{\varkappa\mu}$ are certain functions of φ^ν. Conversely, equations (9) are sufficient in order that the tensor $g_{ij} - -(1-\rho)\bar{\Delta}_{\alpha\beta}\varphi^\alpha_{,i}\varphi^\beta_{,j}$, satisfying the first three conditions, should also satisfy the fourth, i. e. the equations (2). Then the vector N_i takes the shape

$$N_i = \frac{\partial \ln\rho}{\partial x^i}. \tag{10}$$

Equations (9) in a special system of coordinates, where functions φ^ν are the independent variables, give the stratifiable metric (6).

The relation between the values entering equations (6) and (9) is established by the following formulae:

$$g_{\alpha\beta} = \bar{\Delta}_{\alpha\beta}; \quad \frac{\partial T}{\partial\varphi^\beta} = g_{\alpha\beta}T^\beta; \quad \Pi^\sigma_{\alpha\beta} = \Gamma^\sigma_{\alpha\beta} + T^\sigma g_{\alpha\beta}. \tag{11}$$

Values $\Gamma^\sigma_{\alpha\beta}$ are Christoffels for $g_{\alpha\beta}$.

On the metric (6), the geodesic field of pencils, satisfying all of the above four conditions, is defined by tensor a_{ij}, for which we have

$$a_{ij} = g_{ij} - (1-\rho)\, g_{\alpha\beta}\varphi^\alpha_{,i}\varphi^\beta_{,j}, \tag{12}$$

where ρ is defined by the equality

$$2T = \ln\frac{c(\rho-1)}{\rho}$$

when the arbitrary constant c is given.

4. Let us dwell upon some other properties of stratifiable spaces.

I. If, as before, a_{ij} denotes the tensor, defining the geodesic field of pencils characterizing the stratifiable spaces, then from (2) and (10) follows

$$\left(\frac{1}{\rho}a_{ij}\right)_{,k} + \left(\frac{1}{\rho}a_{kj}\right)_{,i} + \left(\frac{1}{\rho}a_{ki}\right)_{,j} = 0.$$

This will mean that the equality

$$\frac{1}{\rho}a_{ij}\frac{dx^i}{ds}\frac{dx^j}{ds} = \text{const}$$

is the first integral of the equations for the geodesic (1). From equalities (6) and (12) il follows that the expression

$$l^{\varkappa T}\frac{\Omega_{pq}dx^p\,dx^q}{ds^2} = \text{const} \tag{13}$$

is also the first integral of these equations.

II. The metric of an n-dimensional Euclidean space, represented in the form (6), takes the shape

$$ds^2 = \sum_\alpha (d\varphi^\alpha)^2 + (\varphi^\nu)^2\,\Omega_{pq}\,dx^p\,dx^q.$$

Here $\Omega_{pq}\,dx^p\,dx^q$ denotes a definite-positive form of constant curvature equal to 1.

The values φ^α may be expressed by Cartesian orthogonal coordinates x^i as follows:

$$\varphi^\alpha = x^\alpha \text{ for } \alpha \neq \nu; \quad \varphi^\nu = \sqrt{(x^\nu)^2 + (x^{\nu+1})^2 + \ldots + (x^n)^2}.$$

III. The necessary and sufficient conditions for the metric (6) to define an Einsteinian space (i. e. $R_{ij}=\lambda g_{ij}$) consist in the equations

$$\left.\begin{aligned}\tilde{R}_{\alpha\beta}&=\lambda g_{\alpha\beta}-(n-\omega)\frac{r_{\alpha\beta}}{r},\\ \tilde{R}_{pq}&=l^{2T}\Omega_{pq}\left[\lambda-\frac{\overset{\alpha}{a}}{r}-(n-\omega-1)\frac{r^{\alpha}r_{\alpha}}{r^{2}}\right],\end{aligned}\right\}\tag{14}$$

where $\tilde{R}_{\alpha\beta}$ and $\tilde{R}_{pq}$ denote Ricci's tensors composed for the tensors $g_{\alpha\beta}$ and Ω_{pq}, respectively, and a new variable r is introduced by the equality:

$$T=\ln r.$$

It may be remarked that for $n=4$ and $\omega=3$, the equality (6) defines the metric of a four-dimensional space of a static field of gravitation ([2]).

5. In conclusion we should like to point out the relation between stratifiable spaces and the space of a gravitation field of a material point.

From the fact that two spatial coordinates of the trajectory of a material particle in the gravitation field of a material point are bound by the relation

$$x^{4}=ax^{3}+b$$

it may be concluded ([4]) that the metric of the respective space-time continuum will have the form (6), if we set $n=4$ and $\omega=2$. A metric of this form for $\Omega_{pq}dx^{p}dx^{q}=d\vartheta^{2}+\sin^{2}\vartheta\,d\varphi^{2}$ is usually obtained on the assumption that the field has a space-spherical symmetry.

Let us give a solution of equations $R_{ij}=\lambda g_{ij}$ for the stratifiable metric (6) for $n=4$, $\omega=2$ and for arbitrary forms $g_{\alpha\beta}d\varphi^{\alpha}d\varphi^{\beta}$, $\Omega_{pq}dx^{p}dx^{q}$.

In this case equations (14) give:

$$r_{,\alpha\beta}=Pg_{\alpha\beta}\tag{15a}$$

and

$$r^{3}\left(\lambda-\frac{r^{\alpha}_{,\alpha}}{r}-\frac{r^{\alpha}r_{\alpha}}{r^{2}}\right)=-K,\tag{15b}$$

where $r_{,\alpha\beta}$ denotes the second covariant derivative of r with respect to the tensor $g_{\alpha\beta}$, and K — the curvature of the form $\Omega_{pq}dx^{p}dx^{q}$, which is a constant number.

From the equations (15) and the conditions of integration, as arise here, there follows two equalities:

$$P=-\frac{m}{r^{3}}+\frac{1}{3}\lambda r \quad\text{and}\quad r^{\alpha}r_{\alpha}=K+\frac{2m}{r}+\frac{1}{3}\lambda r^{2}.$$

For the computed value of P equations (15a) characterize $g_{\alpha\beta}$ as the metric tensor of a definite (up to bending) surface of revolution. It should be noted that the solution has been obtained in the invariant form and, in particular, there is no need to consider the time coordinate separately; with $K=-1$ one can easily pass to the solution of the form given by Schwarzschild.

The first integral of the geodesic of a stratifiable space in the form (13) which can be re-written as

$$r^{2}\frac{d\varphi}{ds}=\text{const}$$

where $d\varphi=\sqrt{\Omega_{pq}dx^{p}dx^{q}}$, takes here the meaning of an integral of areas.

Gorky State University.

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PHYSICS

ON THE SPLITTING OF SACCHAROSE BY ULTRASOUND

By I. L. ROICH and A. S. STARKERMAN

(Communicated by S. I. Wawilow, Member of the Academy, 26. VIII. 1942)

In 1934 a very brief indication on the possibility of splitting of saccharose by means of ultrasound has appeared in a paper by Szalay who claimed to have obtained this result by the way in the course of another investigation. This author asserts that he observed traces of splitting of cane-sugar by means of ultrasound. The sound intensity amounted in his experiments to 10 watt/cm²; the frequency of sound oscillations was equal to 7.22×10^6; a voltage ranging from 30 to 50 kV has been applied to the piezo-quartz plate.

This was the whole information obtained by Szalay, concerning the influence of ultrasound on the inversion of cane-sugar. It enabled Schmidt (1) in his review of the subject to state that Szalay succedeed in obtaining a splitting of cane-sugar by ultrasound.

A somewhat more attentive investigation of the question was carried out by Sokolow (2), who used a 200-watt generator. The voltage applied to the quartz varied between 3 and 12 kV. The frequency was equal to 10^6.

In one of the experiments a concentrated solution of cane-sugar in water was exposed to the action of sound during one hour. In this case no inversion was observed.

In another experiment a solution of saccharose in the presence of nitric acid was exposed to the action of sound. The exposure lasted 72 minutes. That author indicates that during the exposure the temperature did not rise above 16° C. Under these conditions the control solution showed $180.15 \pm 0.27°$, the solution exposed to sound $-180.71 \pm 0.24°$, i. e. the difference amounted to 0.56°, whereas the error of observation was 0.24°.

These data are not sufficiently convincing to enable one to ascribe with certainty the inversion to the action of ultrasound. The observed changes may have been due to difference in temperature. Temperature changes can cause an inversion of the solution of saccharose of vanishingly small magnitude. In the range from 12 to 25° the formula which gives the dependence of the specific rotation on temperature (3) can be used

$$[\alpha]_D^t = [\alpha]_D^{20} - 0.0144\,(t - 20).$$

We have carried out experiments in order to determine the dependence of the rate of change of inversion of saccharose on temperature in the presence of an acid. 40 cm³ of a solution containing 14 g of saccharose were added with 0.5 cm³ of HNO_3 of density 1.4. The solution was divided into two parts, one of which was subjected to heating. Measurements of the rotation angle for both the heated and the control solution, the last one being kept at a constant temperature, were carried out every 30 minutes. The results of measurements are shown graphically in Fig. 1, where the differences of the angles of rotation of the heated and the control solu-

tions, expressed in arbitrary units, are plotted against temperature. This graph shows that in the presence of the acid the magnitude of the inversion of saccharose grows in time considerably with increasing temperature.

Further experiments have been carried out in order to investigate the influence of ultrasound on the splitting of saccharose.

A generator with an ultrasound intensity of 0.25 watt/cm^2 and resonance frequency 6×10^7 has been used.

Aqueous solutions of saccharose with concentrations varying from 0.25 to 40% were exposed to the action of sound. No acid was added. The solution was put into an Ehrlenmeyer thin walled bulb, immersed into water in a brass vessel. To the opening at the bottom of the vessel a piezo-quartz plate was applied. Water served, thus, as a conductor of ultrasound. The cooling was carried out by means of a coil-pipe with running water, placed into the brass vessel. The coil-pipe surrounded the bulb with the solution under investigation. In this way a strict control of temperature, which was kept constant, was secured.

The measurements have not shown any changes in the angle of rotation. As an instance the results of one of the many experiments performed will be given here.

40 cm^3 of the solution contained 14 g of saccharose. The solution was divided into two parts. One part was exposed to sound for 60 minutes. During the exposure the solution was cooled and its temperature was kept at 22.4° C. The saccharimeter of Schmidt showed 10.37 ± 0.05 (arbitrary graduation); the control solution at 22.6° showed 10.36 ± 0.05.

The same solution exposed to sound for 25 minutes (final temperature 32° C) showed 10.33 ± 0.05; the control solution at 32° C showed $10.35 \pm \pm 0.05$. Further experiments of the same kind have been carried out in the presence of nitric acid. We shall give here for the sake of illustration 2 examples from the large number of experiments.

At 11 hours 45 min. a solution of saccharose was prepared, containing 14 g of saccharose + 0.5 cm^3 HNO_3 in 40 cm^3; $t° = 23°$ C. Because measurement of the rotation angle lies outside the possibilities of Schmidt's polarimeter, the measurements were carried out a few hours later.

17 hours 45 min. — initial rotation, 12.24. The solution was exposed to sound during 72 minutes, the measurements were carried out immediately

Table 1

Time	Angle of rotation	
	Exposed solution	Control
18 hrs 58 min.	10.72	10.70
20 » 40 »	9.65	9.65
8 » 30 » (after one day)	0.32	0.32

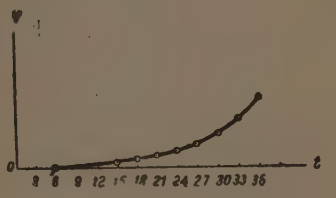

Fig. 1.

after this exposure. It may be seen that no difference whatever in the rotation angles of the two solutions was observed.

Another example: to 30 cm^3 of 10% solution of saccharose 2 cm^3 of nitric acid were added. The solution was divided into two parts. One part was exposed to sound during 1 hour, the other was used for control. The results of measurements are shown graphically in Fig. 2.

The graph shows the trend of the change of inversion with time both for the exposed and the control solution. On the abscissa axis time in hours is plotted, on the ordinate axis — the angles of rotation of the polarization plane in arbitrary units. The curve is a result of measurements of the variation of the inversion with time for the control solution. The

small circles correspond to the data for the exposed solution. Fig. 2 show that the circles exactly coincide with the curve. The angle of rotation attained a constant value in 100 hours. The curve thus shows a complete coincidence of the trend of hydrolysis for both solutions.

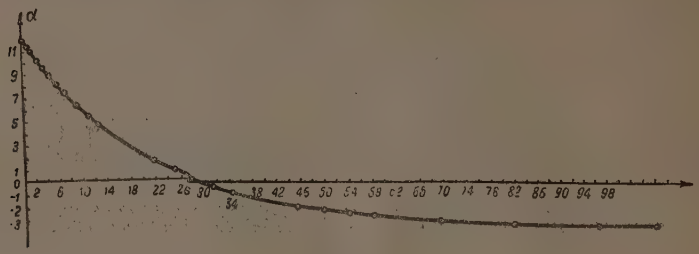

Fig. 2.

The results so far obtained by different authors are listed in Table 2.

Table 2

	Power applied	Frequency	Effect obtained	
			Solution of saccharose	Solution of saccharose $+HNO_3$
Szalay	10 watt/cm²	7.22×10^6	present	—
Sokolow	200 watt	1×10^6	absent	present
Roich and Starkerman	0.25 watt/cm²	6×10^7	absent	absent

It seems that Sokolow gives the power of the generator of electromagnetic oscillations, Szalay—the sound intensity, but without mentioning the method of determination of the power. Whereas according to the opinion of several authors [4] only about 10% of the absorbed power of the electromagnetic oscillations of the generator are transformed into ultrasound, the remaining part being dissipated into Foucault current, dielectrical losses and so on.

We determined the intensity of ultrasound by means of calorimetric measurements, taking into account only the difference of the available sound intensities at the resistance frequency, when a fountain and other phenomena due to ultrasound are observed, and at frequency, differing widely from the resonance one. At any rate the sound intensity, as determined by us, cannot possibly exceed the actual intensity.

From all that was said one can draw the conclusion that at a frequency and power used by us, the splitting of saccharose (either in the presence or in the absence of acid) is not affected by the ultrasound.

In order to form a final opinion on the influence of ultrasound, it would be advisable to carry out experiments at different frequencies and powers. But our choice in this direction was limited by the possibilities of our laboratory.

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PHYSICS

THICKNESS OF LIQUID LAYER ADHERING TO WALLS OF VESSELS ON THEIR EMPTYING AND THE THEORY OF PHOTO- AND MOTION PICTURE FILM COATING

By B. DERJAGUIN

(Communicated by A. N. Frumkin, Member of the Academy, 28. VII. 1942)

Let us examine the stationary state that takes place when an infinite plate inclined to the horizon at an angle α_0 is being taken out of a liquid at a constant velocity U. The liquid entrained by the plate forms a film of thickness h_0. Because the precise determination of this thickness is a matter of considerable difficulty, we shall confine ourselves to such cases when the following method of treating the problem can be applied with sufficient approximation ($\sim 1\%$). Define the X-axis to be directed opposite to the movement of the plate, and the Y-axis perpendicular to this movement (see figure). Let h_0 be the value of h at $x = -\infty$. Suppose that a thickness $\delta \gg h_0$ may be found such that for $h > \delta$ the surface of the liquid would satisfy the equations of capillary statics. At the same time, the condition $\left|\frac{dh}{dx}\right| \ll 1$ should be fulfilled for $h \leqslant \delta$. Then the surface of the liquid can be obtained by fitting together the static capillary surface undisturbed by the movement of the plate with the surface of the film adjoining the wall which has a slowly varying thickness $h = h(x)$. If U is small enough, the results of this operation will be sufficiently accurate.

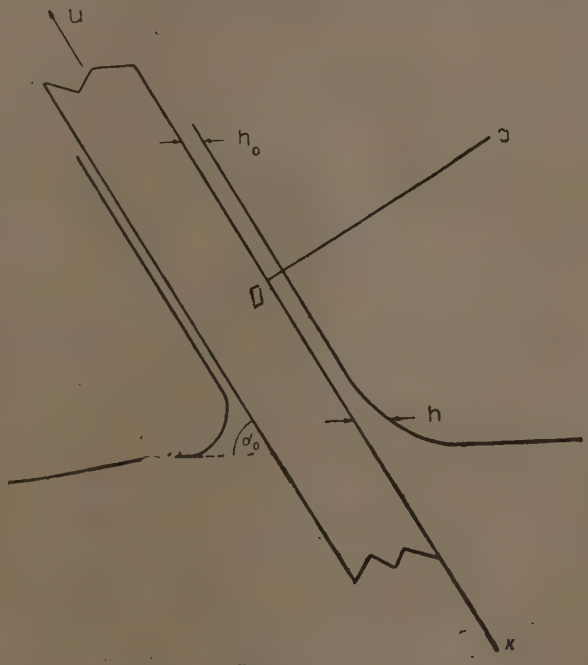

Fig. 1.

The smallness of U permits us, moreover, to neglect inertia as compared to viscosity. The streamlines of the liquid become narrower in the region

of the thin film and, as the forces of inertia are small, there is a consequent increase of pressure just as in the case of wedging-action, known in the hydrodynamic theory of lubrication, which offsets the capillary pressure of the meniscus of the liquid *. In the zone of the thin layer the hydrodynamical equations for the stationary case give:

$$\frac{\partial p}{\partial x} - \rho g \sin \alpha_0 = \eta \frac{\partial^2 u}{\partial y^2}, \tag{1}$$

where p is pressure; ρ, density; g, acceleration of gravity; η, viscosity; u, velocity of the liquid's particles. The boundary conditions are:

$$u = -U \quad \text{for} \quad y = 0$$
$$\frac{\partial u}{\partial y} = 0 \quad \text{for} \quad y = h.$$

Furthermore, the I capillarity law of Laplace gives:

$$\sigma \frac{\partial^2 h}{\partial x^2} + p = \text{const}, \tag{2}$$

where σ is the surface tension of the liquid. Putting $\frac{\partial^2 p}{\partial y\, \partial x} \cong 0$, we find from (1) and (2) that:

$$\sigma \frac{d^3 h}{\partial x^3} + \rho g \sin \alpha_0 = 3\eta \left[\frac{U}{h^2} - \frac{Q}{h^3} \right], \tag{3}$$

where $Q = -\int_0^h u dy$ is proportional to the discharge of liquid per second.

It may easily be shown that

$$Q = U h_0 - \frac{\rho g \sin \alpha_0}{3\eta} h_0^3. \tag{4}$$

Introducing further the parameters $\beta = \frac{\rho g \sin \alpha_0 h_0^3}{3\eta U}$ and $\gamma = \frac{3\eta U}{\sigma}$, and the variables $\frac{h}{h_0} = H$ and $\gamma^{\frac{1}{3}} \frac{x}{h_0} = \chi$, we get from (3) and (4)

$$\frac{\partial^3 H}{\partial \chi^3} + \beta = \frac{1}{H^2} - \frac{(1-\beta)}{H^3} = f(H). \tag{5}$$

For $-\chi \gg 1$ and $(H-1) \ll 1$ the solution of (5) takes the form:

$$H = 1 + A e^{(1-3\beta)^{\frac{1}{3}} \chi}, \quad (A = \text{const}).$$

Multiplying (5) by $\frac{d^2 H}{d\chi^2} d\chi$ and integrating over an interval from $\chi = -\infty$ to the zone corresponding to $h = \delta$, $\chi = \varepsilon$, we obtain

$$\frac{1}{2} \left(\frac{d^2 H}{d\chi^2} \right)^2 + \beta \left(\frac{dH}{d\chi} \right) = \int_{-\infty}^{\varepsilon} f(H) \frac{d^2 H}{d\chi^2} d\chi \approx \int_{-\infty}^{+\infty} f(H) \frac{d^2 H}{d\chi^2} d\chi = B(\beta). \tag{6}$$

On the other hand, it may be readily shown that all cylindrical capillary surfaces (with horizontal generatrices) satisfy the relation

$$\frac{\sigma}{2\rho g} \frac{1}{R^2} + \cos \alpha = C, \tag{7}$$

where R is the radius of curvature; α, the angle of inclination of the tangent plane to the horizon at any point of the surface; C is a parameter characterizing the particular surface of the family.

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* If h is about 10^{-5} cm or less, part, if not all of the capillary pressure may be counterbalanced by an equilibrium disjoining action, which we have investigated in a series of papers (1).

In the zone near $\chi = \varepsilon$ instead of (7) we can, by passing to the variables H and χ, approximately write:

$$\frac{1}{2}\left(\frac{d^2H}{d\chi^2}\right) + \beta\left(\frac{dH}{d\chi}\right) = \frac{C - \cos\alpha_0}{\sin\alpha_0}\,\frac{\beta}{\gamma^{\frac{1}{2}}}. \qquad (7')$$

The condition for «fitting together» is

$$B(\beta) = \int_{-\infty}^{+\infty} f(H)\,\frac{d^2H}{d\chi^2}\,d\chi = \left[\frac{C - \cos\alpha_0}{\sin\alpha_0}\right]\frac{\beta}{\gamma^{\frac{1}{2}}} = \frac{\beta}{\theta}, \qquad (7'')$$

where $\theta = \frac{\sin\alpha_0}{(C - \cos\alpha_0)}\,\gamma^{\frac{1}{2}}$.

Using an electro-integrator of Gutenmacher for approximate integration* of equation (5) or, strictly speaking, for the extraction from it of $\frac{dH}{d\chi}$ as a function of H, one can by means of (7″) plot β against θ. Through the integration of (5) it is possible a posteriori to verify whether the condition $\left|\frac{dh}{dx}\right| \ll 1$ may be fulfilled at the point where $h = \delta$, and at the same time the necessary precision secured by choosing a high enough value of ε. The actual precision falls with increasing β (or θ), and grows to infinity as β approaches 0. Therefore small values of β and θ are of particular interest (see below).

For h_0 we find

$$h_0 = \sqrt{3\beta(\theta)\,\frac{\eta U}{\rho g \sin\alpha_0}}, \quad \theta = \frac{\left[\frac{3\eta U}{\sigma}\right]^{\frac{1}{3}} \sin\alpha_0}{C - \cos\alpha_0} = R_0^2\,\frac{2\rho g \sin\alpha_0}{\sigma}\left(\frac{3\eta U}{\sigma}\right)^{\frac{1}{3}}. \qquad (8)$$

In practice, we have always to deal with plates of finite height; such plates, when being taken out, have a film formed on them, the thickness of which, owing to flowing down of the liquid, decreases to zero in the direction to the upper edge of the plate; h_0 corresponds to the lower portion of the layer. Of major interest is the mean or effective thickness $\bar{h}_0 = \frac{Q}{U}$:

$$\bar{h}_0 = \frac{Q}{U} = \sqrt{3\beta(1-\beta)^2\,\frac{\eta U}{\rho g \sin\alpha_0}}. \qquad (8')$$

The thickness of a layer which has cooled into a gel and lost fluidity, as in the case of emulsion coating of photographic films, is also expressed by $\bar{h}_0$.

In the case of an infinite liquid surface $C = 1$. In the case of capillary surface between two parallel vertical plates C is determined by the distance between the plates, x, from a transcendental equation, where it enters the modulus of complete elliptical integrals.

Limiting Laws. With $\gamma \gg 1$ the adopted method of calculation becomes quite inapplicable. Its formal application, however, when $\gamma \to \infty$ gives a value of $\beta = \frac{1}{3}$ leading to the formula:

$$\bar{h}_0 = \sqrt{\frac{4}{9}\,\frac{\eta U}{\rho g \sin\alpha_0}} = \sqrt{c\,\frac{\eta U}{\rho g \sin\alpha_0}}. \qquad (9)$$

This formula has been formerly established by me** on other considerations.

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* This part of the work has been carried out by G. Tolstov, Eng. D., to whom thanks are due.

** Reports delivered at the Institute for Photo- and Cinematographic Research in 1939 and 1940 and at the Institute for Colloid- and Electrochemistry of the Academy of Sciences of the USSR. A similar formula with $c = \frac{2}{9}$ was deduced earlier by other authors (2) from quite an arbitrary and erroneous boundary condition.

At the same time the dependence of β upon θ accounts theoretically for the decrease of c to ~ 0.1 and lower with decreasing U and, accordingly, with decreasing γ, which fact has been discovered by L. A. Konovalova at the Institute for Photo-and Cinematographic Research and by V. P. Lazarew and N. N. Zachovaeva in our laboratory.

As has been pointed out above, function $\beta(\theta)$ is the more accurately determined by the method of fitting together, the smaller θ. It is seen from (7″) that at the limit $\beta = B(0)\theta$, where $B(0) = B_0$ is a constant which has been found by computation to equal 0.203... The corresponding limiting expression for h_0 when $U \to 0$ is obtained in a general form, by putting $\beta = B_0\theta$ in (8) and $\beta = 0$ in (5):

$$h_0 = \overline{h}_0 = \sqrt{2B_0}R_0\left(\frac{3\eta U}{\sigma}\right)^{\frac{1}{3}} \qquad (10)$$

or, in the case of a vertical wall and an infinite surface of the liquid*

$$h_0 = \overline{h}_0 = k\frac{(\eta U)^{\frac{1}{3}}}{(\rho g)^{\frac{1}{3}}\sigma^{\frac{1}{3}}} \qquad (10')$$

where

$$k = 3^{\frac{1}{3}}\sqrt{B_0} = 0.938... \qquad (10'')$$

Note 1. If the described computations are to be applied to the measurement of the volume of the liquid remaining on the walls of cylindrical burettes, allowance should be made for the curvature of the walls (of radius r). For $\frac{h}{r} \ll 1$ the whole set of equations (1)—(6) can be left without change in a first approximation. It is easily seen that the formula (10) for the limiting law can also be preserved without modification, if R_0 is defined to mean the radius of curvature of the meridional section of the capillary surface (near the «waterline»).

Note 2. The dissipation of energy per second D in the film entrained by the plate is proportional to $\rho g(Uh_0 - Q) = \frac{\rho^2 g^2 h_0^3}{3\eta}$. Assuming that in this case, too, $D \sim U^2$ for small U and taking into account the dimensionality, we for $C = 1$, $\alpha_0 = \frac{\pi}{2}$ arrive at formula (10′).

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* The limiting value for h_0 when $U \to 0$ was somewhat earlier obtained by V. G. Levich and L. D. Landau by discarding the small term (the term with g) in the initial equation; of this we have been kindly informed by the former author. But, as the present author was the first to outline (private communication to V. G. Levich) the method for the solution of this problem [including equations (3) and (4), and the idea of fitting together the solution of equation (3) with the solution of the problem of capillary statics],—the result obtained can justly be considered a fruit of a collective effort.

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GEOPHYSICS

TEMPERATURE DISTRIBUTION IN THE ATMOSPHERE OF THE EARTH

By I. A. KIBEL

(Communicated by N. E. Kochin, Member of the Academy, 3. XII. 1942)

The equation of the heat inflow in the atmosphere of the Earth may with a high degree of accuracy be written in the form:

$$c_p \rho \frac{dT}{dt} = A \frac{dp}{dt} + \varepsilon_1 + \varepsilon_2 + \varepsilon_3. \qquad (1)$$

Here T is temperature; p, pressure; ρ, density; t, time; c_p, specific heat at a constant pressure; A, thermal equivalent of work; $\frac{d}{dt}$, symbol of the individual derivative with respect to time; ε_1, heat inflow (per unit volume and unit time) due to radiation; ε_2, heat inflow due to turbulent heat conductivity; ε_3, heat inflow from condensation or evaporation. In most cases this single equation proves insufficient for temperature determination; and it is not unless one combines it with the equations of motion and continuity that the system can be «closed». To the small number of problems that can be solved by using equation (1) alone, belongs in particular that of the mean annual temperature distribution in space. The present article is devoted to this problem.

In this case it is possible to leave out of consideration the velocity field (the mean velocity being equal to zero) and to regard pressure as independent of (slightly dependent on) time. The left-hand side of equation (1) will then contain the term $c_p \rho \frac{\partial T}{\partial t}$ only; now, if we consider the mean T as independent of time, we simply obtain $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$. The magnitude ε_3 can be neglected. For writing ε_1 let us dwell upon the primitive scheme by Schwarzschild-Emden (under the assumption that there exist only two flows of radiation: one proceeding from above, including the long-wave flow A and the short-wave flow S from the Sun, and another, proceeding from below, the long-wave flow B). Then

$$\varepsilon_1 = \alpha \rho_w (A + B) + \alpha' \rho_w S - 2\alpha \rho_w f E,$$

where α, α' and αf are the absorption coefficients for the «gray» radiation (α and αf for the long-wave, α' for the short-wave radiation); ρ_w is density of the radiation absorbing matter which largely consists in water vapour; $E = \sigma T^4$ (σ — Stephan-Boltzmann's constant). The factor f used here as a function of temperature, has been introduced according to Hulbert (1). Further, ε_2 has the shape:

$$\varepsilon_2 = \frac{\partial}{\partial x}\left(\lambda' \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda'' \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right),$$

where x, y are horizontal coordinates; z is the vertical one; λ', λ'', λ are coefficients of turbulent heat-conductivity, corresponding to those axes. The equations to be written for A and B are, as always,

$$\frac{\partial A}{\partial z} = \alpha\rho_w (A - fE) \tag{2}$$

$$\frac{\partial B}{\partial z} = \alpha\rho_w (fE - B) \tag{3}$$

$$\frac{\partial S}{\partial z} = \alpha\rho_w S. \tag{4}$$

We shall consider here the case of purely-vertical mixing. Equation (1) will then ultimately assume the form:

$$0 = \frac{d}{dz}\left(\lambda \frac{dT}{dz}\right) + \alpha\rho_w (A + B - 2fE + \beta S), \tag{5}$$

where $\beta = \frac{\alpha'}{\alpha}$. With $z = \infty$ and axis z being directed vertically upwards, the following boundary conditions will be adopted:

$$A = 0 \tag{6}$$

(near the limits of the atmosphere solar radiation only falls from above),

$$S = W. \tag{7}$$

Allowance is made here for the reflection of solar rays from clouds or the Earth surface so that W is the mean quantity of the radiant energy of the Sun, multiplied by Γ (where Γ is the mean albedo),

$$B = W \tag{8}$$

(the atmosphere gives off as much energy as it receives).

Let us now adopt the following boundary conditions on the Earth for $z = 0$:

$$-\lambda \frac{dT}{dz} + \bar{\lambda} \frac{d\bar{T}}{dz} = A + S - B, \tag{9}$$

$$B = qE \tag{10}$$

(λ, $\bar{T}$ are the coefficient of heat conductivity and the soil temperature, respectively; the introduction of q points to the fact that the Earth does not emit radiation as an absolutely black body). The appearance of $\bar{T}$ makes us add one equation more: we do not write it here, since in the problem dealing with the annual mean value of T the term $\bar{\lambda} \frac{d\bar{T}}{dz}$ in the boundary condition can and will be neglected.

It is convenient to look for E rather than T, and instead of A and B, for their difference and sum. By introducing the dimensionless magnitudes ε, a, y from the equalities

$$fE = W\varepsilon, \quad B + A = Wa, \quad B - A = Wy, \tag{11}$$

the dimensionless independent variable τ (optical «thickness» for the gray radiation)

$$\tau = \int_z^\infty \alpha\rho_w dz, \tag{12}$$

and taking notice of the fact that according to (4) and (7)

$$S = We^{-\beta\tau}, \tag{13}$$

the problem will easily be reduced to the integration of a single equation

$$\frac{d^2y}{d\tau^2} - \left(1 + \frac{8\sigma T^3}{\lambda a \rho_w}\right) y = -\frac{8\sigma T^3}{\lambda a \rho_w} e^{-\beta\tau} \tag{14}$$

under boundary conditions:

$$\text{for } \tau = 0: \quad y = 1 \tag{15}$$

$$\text{for } \tau = \tau_0: \quad y + \frac{q}{f} y' = \left(\frac{q}{f} - 1\right)\left(1 + \int_0^{\tau_0} y\, d\tau\right) \tag{16}$$

where

$$y' = \frac{dy}{d\tau}, \quad \tau_0 = \int_0^{\infty} a\rho_w\, dz. \tag{17}$$

After y has been defined, magnitudes a and ε are found from the formulae

$$a = 1 + \int_0^{\tau} y\, d\tau, \quad \varepsilon = \frac{1}{2}\left(1 + \int_0^{\tau} y\, d\tau - \frac{dy}{d\tau}\right). \tag{18}$$

The magnitudes T, ρ_w, λ and α vary with the altitude. The value of the combination $\frac{fT^3}{\lambda a \rho_w}$ remains, however, almost constant*. Thus, if we simply put $\frac{fT^3}{\lambda a \rho_w} = \text{const}$, and introduce into this equality T, α and ρ_w from empirical data, a slight increase of λ with altitude will be obtained.

Let us now simply assume that

$$\frac{4\sigma T^3 f}{\lambda a \rho_w} = \frac{m^2 - 1}{2} = \text{const}. \tag{19}$$

Equation (14) then will give

$$y = c_1 e^{m\tau} + c_2 e^{-m\tau} + \frac{m^2 - 1}{m^2 - \beta^2} e^{-\beta\tau}, \tag{20}$$

whereby c_1 and c_2 are determined from (15) and (16). The elementary calculations become still simpler, owing to the fact that τ_0 practically uses to be greater than unity (of the order of 10), so that ($m > 1$) the terms of the form $e^{-m\tau_0}$ are very small as compared to unity, and we have

$$c_1 \approx \frac{\frac{m^2 - 1}{m^2 - \beta^2}\left[(q-1)\frac{1 - e^{-\beta\tau_0}}{\beta} - (1 - q\beta)\, e^{-\beta\tau_0}\right] + (q-1)\left(1 + \frac{1 - \beta^2}{m^2 - \beta^2}\frac{1}{m}\right) e^{-m\tau_0}}{1 + mq + \frac{1-q}{m}} \tag{21}$$

$$c_2 \approx \frac{1 - \beta^2}{m^2 - \beta^2}. \tag{22}$$

*Apart from some dozen meters near the Earth.

Thus, ultimately

$$\varepsilon = \frac{1}{2}\frac{m^2-1}{m^2-\beta^2}\left\{\frac{m^2-\beta^2}{m^2-1}+\frac{1-\beta^2}{m^2-\beta^2}\frac{1}{m}+\right.$$

$$\left.+\beta e^{-\beta\tau}+\frac{1-e^{-\beta\tau}}{\beta}+\frac{1-\beta^2}{m}e^{-m\tau}-\frac{m^2-\beta^2}{m}c_1 e_{m\tau}\right\}. \qquad (23)$$

As a particular case, with $\lambda = 0$ (absence of turbulence) and $\beta = 0$ (full transparency to solar radiation) we once more obtain from (23) Emden's formula:

$$\varepsilon = \frac{1+\tau}{2}.$$

For a concrete calculation the author has chosen a latitude of 42° and put $m = 1.75$, $\beta = \frac{\alpha'}{\alpha} = 0.2$, $\frac{q}{f} = 1.15$, $W = 0.138$ g-cal./cm² min*. According to Hann's empirical law, τ is determined from the formula

$$\tau = \int_z^\infty \alpha_0 \rho_{w_0} T_0 \frac{10^{-\frac{z}{l}}}{T}\, dz, \qquad (24)$$

where $l = 8\times 10^5 \text{cm}^{-1}$; α_0, ρ_{w_0}, T_0 are values of α, ρ, T for the Earth, respectively. For sake of simplicity of calculation, T was taken in this integral according to Emden: $\frac{T}{T_\infty} = (1+\tau)^{\frac{1}{4}}$, so that computation was done by means of the formula

$$\frac{4}{5}\left[(1+\tau)^{\frac{5}{4}}-1\right] = \alpha_0 \rho_{w_0}\frac{T_0}{T_\infty}\frac{l}{\ln 10}10^{-\frac{z}{l}}. \qquad (25)$$

For α_0, ρ_{w_0} I took the mean values ($\rho_{w_0} = 6.2\times 10^{-6}$ g/cm³, $\alpha_0 = 7.25$ cm²/g) and $\frac{T_0}{T_\infty}$ was put equal to $\frac{220}{280}$. In this way one obtains $\tau_0 = 12.6$.

For determining the quantity $W\varepsilon = \sigma f T^4$, I have obtained the expression

$$\sigma f T^4 = 0.047\left[6.80-4.80e^{-0.2\tau}+0.57e^{-1.75\tau}-0.36e^{1.75(\tau-12.6)}\right]. \qquad (26)$$

Finally, to compute T, f was taken from Hulbert's plot and there was constructed a table of values of T as a function of $\sigma f T^4$, which is partly given below.

Table 1

T (°C)	210	220	230	240	250	260	270	280	290	300	310	320
$\sigma f T^4$ $\frac{\text{g-cal.}}{\text{cm}^2\,\text{min}}$	0.106	0.124	0.145	0.167	0.192	0.219	0.249	0.284	0.322	0.365	0.410	0.462

The results of calculation of T are also tabulated (the temperatures are everywhere rounded off up to 0.5°C). For sake of comparison, along with these values are given the actual means of temperature, taken at the same altitudes, as plotted from radioprobings (³) effected within 1938, 1939 and 1940 at the station Omaha, USA (Table 2).

* According to Milankovich(²), the inflow of heat from the Sun at a latitude of 42°, as averaged for a year, takes up about $\frac{2}{\pi}$ 0.72 g-cal./cm² min. We assume the mean albedo for a year to be $\Gamma = 0.7$, hence the figure $W = 0.138$ g-cal./cm² min.

Table 2

z, km	0.3	0.5	1.0	2.0	3	4	5	6
τ	12.6	12.0	10.5	8.4	6.5	5.1	4.0	3.2
T from (27)	8	9	9	4.5	—0.5	—6.5	—13.0	—19.5
T_{actual}	8.1	9.1	8.9	5.2	—0.1	—6.1	—12.2	—19.5

z, km	7	8	9	10	11	12	13	∞
τ	2.5	1.9	1.45	1.1	0.8	0.6	0.4	0
T from (27)	—26.5	—33.5	—40.0	—45.0	—49.0	—52.0	—53.0	—55.0
T_{actual}	—26.4	—33.7	—40.8	—47.15	—52.35	—55.7	—57.7	—

The «stratosphere» begins at a due altitude, but it is everywhere warmer than in reality. However, the figure —55° obtained by us approaches the figures that usually are adopted for this latitude. The disagreement may possibly be due to the fact that the number of observations carried out at high altitudes was to small, and a period of three years is insufficient to find a mean; but it is more likely that formulae (24) and (25) are not accurate enough for large z. It should be noted that the term containing c_1 has an effect in (23) within the lowest kilometer only.

In order to describe more precisely the behaviour of the temperature in the lower layers of the atmosphere (of the order of some ten meters), the coefficient $\frac{4\sigma T^3 f}{\lambda a \rho_w}$ can be assumed there to be variable; for this it is sufficient to consider λ as a linear function of height, and the rest of the quantities, entering into this coefficient, as constants. The solutions for the lower layer can be obtained in the form of series, whereby the artibrary constants of integration must be selected under the condition that y and $\frac{dy}{d\tau}$ corresponding to the two solutions coincide at the boundary of transition from a variable λ to $\lambda = \text{const}$. These calculations can be taken as a basis when determining inversions from data obtained at the Earth surface.

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CHEMISTRY

KINETICS OF CONSECUTIVE REACTIONS CONSISTING OF BIMOLECULAR AND MONOMOLECULAR STAGES

By A. A. BALANDIN and L. S. LEIBENSON

(Communicated by V. G. Chlopin, Member of the Academy, 5.XII. 1942)

Consecutive reactions involving bimolecular stages are of practical importance (such is polymerization, for instance). This notwithstanding, the solution of their kinetic differential equations, contrary to the case of complex monomolecular reactions (1), has been seldom effected (2) on account of the mathematical difficulties involved. Usually only the stationary case is examined (3), where considerable simplification is reached by using Bodenstein's rule, according to which the sum of formation and decomposition velocities is equal to zero for every intermediate product.

Below we examine a succession of transformations

$$2A \rightarrow B \rightarrow C \qquad (1)$$

which presents a characteristic case of a successive reaction starting with the bimolecular stage. Let us show that by using the integral logarithm (4) the kinetic equations can be solved in this case.

A, B, C denote the concentrations of substances A, B, C; k_1 and k_2 are velocity constants of the first and second stage, respectively. Scheme (1) leads to the following system of kinetic differential equations:

$$\frac{dA}{dt} = -2k_1A^2; \quad \frac{dB}{dt} = k_1A^2 - k_2B; \quad \frac{dC}{dt} = k_2B, \qquad (2)$$

with the initial conditions

$$\text{at } t=0: \quad A=A_0, \; B=C=0. \qquad (3)$$

Hence

$$\frac{d}{dt}(A/2+B+C)=0 \qquad (4)$$

and, after integration,

$$A/2+B+C=A_0/2. \qquad (5)$$

We shall first employ the ordinary approximate methods, and afterwards give the full solution of equations (2).

Stationary State, $dB/dt=0$. The second of the equations (2) transforms into

$$k_2B=k_1A^2. \qquad (6)$$

After substitution in (5), we have

$$\frac{k_1}{k_2}A^2+\frac{1}{2}A=\frac{1}{2}A_0-C. \qquad (7)$$

Depending upon the ratio between the velocity constants k_1 and k_2, the first or second term in (7) can be neglected and, taking into account equations (2) and (6), we obtain two limiting cases:

$$\text{a)}\quad \frac{\sqrt{dC/dt}}{\frac{A_0}{2}-C}=2\sqrt{k_1} \qquad (8) \qquad \text{at } \frac{k_2}{2k_1}\gg A, \qquad (9)$$

$$\text{b)}\quad \frac{dC/dt}{\frac{A_0}{2}-C}=k_2 \qquad (10) \qquad \text{at } \frac{k_2}{2k_1}\ll A. \qquad (11)$$

If one time $\sqrt{dC/dt}$ and another time dC/dt be plotted against C (whereby a concentration curve is obtained) the result will be two straight lines crossing the abscissa at a distance of $A_0/2$ from the origin. As a matter of fact, concentration curves of the type (a) and of the type (b) are curves with maxima, because they include also the origin.

General Case. The full solution of equations (2) is

$$\left.\begin{aligned} A&=\frac{A_0}{1+2k_1A_0t},\\ 2B&=A_0e^{-k_2t}-A-J,\\ 2C&=A_0(1-e^{-k_2t})+J, \end{aligned}\right\} \qquad (12)$$

where

$$J=he^{-\frac{h}{A}}\left(\operatorname{li}e^{\frac{h}{A}}-\operatorname{li}e^{\frac{h}{A_0}}\right); \qquad (13)$$

li denotes the integral logarithm, and

$$h=\frac{k_2}{2k_1}. \qquad (14)$$

Function $\overline{Ei}(x)=\operatorname{li}e^x$ has been tabulated.

When the reaction is complete, $t\to\infty$, the equations (12) in accordance with experiment give

$$A\to 0,\ B\to 0,\ C\to A_0/2, \qquad (15)$$

as in this case $J\to 0$:

$$\lim_{t\to\infty}\left[\frac{k_2A_0}{2k_1}e^{-k_2t}\int_0^t\frac{e^{k_2t}}{1+2k_1A_0t}dt\right]=0, \qquad (16)$$

where the value of J is given in brackets.

Unlike the stationary case, the equations (12) make it possible to find the whole course of both the kinetic curves $dC/dt=f(t)$ and the concentration curves $dC/dt=f(C)$ (remember that $dC/dt=k_2B$).

Particular Cases. The limiting cases (a) and (b) mentioned above are obtained from equations (12) when some of their terms may be neglected.

a) Put

$$B=0 \qquad (17)$$

which is approximately fulfilled, if there is no B at the beginning of the reaction, and if during the reaction B is more rapidly decomposing than forming.

Substituting in (5) equation (17) and A from (12), we obtain

$$\frac{A_0}{2}-C=\frac{1}{2}\frac{A_0}{1+2k_1A_0t}, \qquad (18)$$

and, after differentiation,

$$\frac{dC}{dt}=\frac{k_1A_0^2}{(1+2k_1A_0t)^2}. \qquad (19)$$

(18) and (19) give (8).

b) Put

$$J = 0. \qquad (20)$$

Then the third of the equations (12) gives

$$\frac{A_0}{2} - C = \frac{A_0}{2} e^{-k_2 t}, \qquad (21)$$

and, after differentiation,

$$\frac{dC}{dt} = \frac{k_2 A_0}{2} e^{-k_2 t}. \qquad (22)$$

Dividing (22) by (20), we at once obtain the equation (10).

Comparison with Experiment. Concentration curves with maxima of the types (a) and (b) have recently been observed experimentally in the case of methane formation from equimolecular mixtures of sodium acetate and sodium hydroxide in melts[5]. The first, bimolecular stage presents the formation of an intermediate product, the acid or the acetate, whereas during the second, monomolecular stage this product decomposes into methane and sodium carbonate. In the light of the results reported in this paper, the experimental data may be interpreted as follows.

The curves for the pure mixtures of initial substances belong to type (a). This must be taken to mean that: 1) the reaction is really a successive one; 2) its first stage is bimolecular; 3) the velocity constant of the formation of the intermediate product is by far less than that of its convertion into ultimate products; 4) the concentration of the intermediate product is low, and 5) the velocity constants and the activation energy as measured from the slope of the straight portions of the concentration curves belong in fact to the first bimolecular stage above-mentioned. Examination of the stationary case made in a quite analogous manner shows that if the first stage in scheme (1) were reversible, equation (8) would remain unaltered but for $\sqrt{k_1}$ being substituted with $\sqrt{k_1 k_2/(k_2+k_3)}$, or, in other words, k_1 would be multiplied by the probability of the decomposition of B into C, if the probability of the decomposition of B into A or C is equal to unity.

In the presence of certain catalyzers, or of an excess of soda, the reaction investigated yields kinetic curves of the type (b), which testifies to the fact that the ratio of the velocity constants is changed towards a relative increase of the first stage constant, and what has been stated above concerning the curves of type (a) may be said, mutatis mutandis, to be true of these curves, too.

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CHEMISTRY

SOME HOMOLOGUES OF CYCLOPENTYL- AND CYCLOHEXYL-BENZENE AND PRODUCTS OF THEIR HYDROGENATION

By E. S. POKROVSKAYA

(Communicated by S. S. Nametkin, Member of the Academy, 9. VII. 1942)

The successful development of methods devised for the analysis of complex hydrocarbon mixtures, such as kerosine and lubricating oil fractions, must inevitably be founded on the knowledge of the properties of various high molecular hydrocarbons.

For some time past a number of papers on this subject has been published (1). These papers were, however, concerned chiefly with isoparaffines and naphthenes and aromatics containing aliphatic side chains. Data upon hydrocarbons constituted of aromatic and naphthenic rings in combination with aliphatic radicals remain still insufficient. Some hydrocarbons of this type were described in my previous paper (2). In the present work a number of hydrocarbons have been prepared, falling into the temperature range of kerosine and diesel oil fractions, that is hydrocarbons of the series of cyclopentylmesithylene, isopropylcyclopentyl-benzene, cyclopentyl-paracymene and cyclohexyltoluene, together with some corresponding naphthenes.

The said hydrocarbons were synthesized by condensing mesithylene, isopropylbenzene and para-cymene with cyclopentene and toluene with cyclohexene in the presence of aluminium chloride. The elementary constitution of all the hydrocarbons was determined. Their properties are listed in the table (Nos. 1—6).

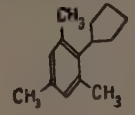
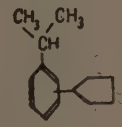
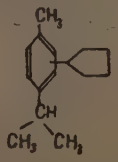
The critical aniline points of all the hydrocarbons were found below 15°. In the same table are given some hydrocarbons that I have prepared previously (Nos. 7—9).

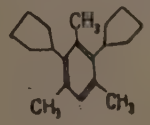
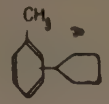
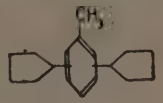
After repeated fractionation mesithylene had a boiling point 164.5° at 760 mm, $d_4^{20} = 0.8642$, $n_D^{19.8} = 1.4988$ and molecular refraction $MR = 40.77$ (theoretical $MR = 40.16$). Its alkylation with cyclopentene was carried out in an ice-bath. After a careful fractionation, from the products of reaction there were isolated cyclopentylmesithylene (No. 1) and dicyclopentylmesithylene (No. 4).

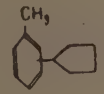
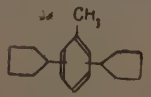
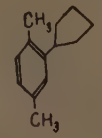
For the synthesis of isopropylcyclopentyl-benzene isopropyl-benzene with a b. p. 151.5—152° by 744 mm, $d_4^{20} = 0.8613$ and $n_D^{20} = 1.4912$ was employed.

A preliminary experiment on treatment of this hydrocarbon with 10% of aluminium chloride has shown that it is easily decomposed and converted into 5.2% benzene, 3.9% toluene and 5.7% diisopropylbenzene. Consequently, the time of alkylation with cyclopentene was reduced to 3 hours, and the quantity of aluminium chloride employed, to 2.8%. From the products of this reaction isopropylcyclopentyl-benzene was isolated (No. 2).

Para-cymene had a boiling point 176.6—176.8° at 759 mm, $d_4^{20} = 0.8568$ and $n_D^{20.5} = 1.4912$. After treatment with 6% of aluminium chloride during

No.	Name and formula	B. p. in °C	M. p. in °C	d_4^{20}	n_D^{20}	MR found	MR calcd.	Viscosity in centistoks 10°	20°	50°	70°	100°
1	Cyclopentylmesithylene  $C_{14}H_{20}$	276—276.5 767 mm	below—50°	0.9435	1.5325	63.77	61.04	8.87	6.26	—	2.08	1.39
2	Isopropylcyclopentyl-benzene *  $C_{14}H_{20}$	266,5—267,5 757 mm	below—50°	0.9222	1.5190	61.93	61.05	4.82	3.76	2.11	1.58	1.11
3	Cyclopentyl-para-cymene *  $C_{15}H_{22}$	121—122 6.5 mm	below—50°	0.9227	1.5219	66.81	65.67	8.87	6.26	2.94	2.05	1.29

No.	Name and formula	B. p. in °C	M. p. in °C	d_4^{20}	n_D^{20}	*MR* found	*MR* calcd.	Viscosity in centistoks 10°	20°	50°	70°	100°
4	Dicyclopentylmesithylene  $C_{19}H_{28}$	164—165 1 mm	below—50°	0.9870	1.5527	83.03	81.13	—	258.92	—	12.70	5.39
5	Cyclohexyltoluene*  $C_{13}H_{18}$	261—262 762 mm	below—50°	0.9353	1.5250	57.06	56.43	5.52	4.05	2.17	1.58	1.11
6	Dicyclohexyltoluene*  $C_{19}H_{28}$	165—166 3 mm	—3 glass	0.9645	1.5378	83.12	81.94	—	553.33	38.45	14.27	5.56

No.	Name and formula	B. p. in °C	M. p. in °C	d_4^{20}	n_D^{20}	MR found	MR calcd.	Viscosity in centistoks 10°	20°	50°	70°	100°
7	Cyclopentyltoluene (2)  $C_{12}H_{16}$	237.5 757 mm	below—50°	0.9401	1.5287	52.34	51.51	2.82	2.29	1.41	—	—
8	Dicyclopentyltoluene (2)  $C_{17}H_{24}$	150—152 2 mm	below—50°	0.9694	1.5391	73.75	72.70	24.75	15.70	5.70	—	2.14
9	Cyclopentyl-para-xylol (2)  $C_{13}H_{16}$	254.5—255 762 mm	—44° crystal	0.9414	1.5291	56.37	56.42	4.55	3.52	1.94	—	—

* The position of the naphthenic ring has not been determined.

4 hours in an ice-bath almost all the product boiled at 174—177° and had $n_D^{20} = 1.4919$.

One can therefore hold for certain that practically the hydrocarbon has remained unchanged. Its alkylation with cyclopentene was carried out under conditions described above. The principal product of reaction was cyclopentyl-para-cymene (No. 3).

The hydrocarbons of the series of cyclohexyltoluene were synthesized for the purpose of comparing them with hydrocarbons of the series of cyclopentyltoluene that have been prepared previously [2]. Toluene had a b. p. of 110°C at 768 mm and $n_D^{17.5} = 1.4984$. It has previously been found [2] that treatment of toluene with aluminium chloride in an ice-bath is not followed by change of structure. As a result of its alkylation with cyclohexane cyclohexyltoluene (No. 5) and dicyclohexyltoluene (No. 6) were obtained.

Cyclohexyltoluene has already been described by Bodroux [3] and Tzukervanik and Sidorova [4]; these authors have also obtained dicyclohexyltoluene as a crystalline product with a melting point 93.5°.

The hydrocarbons Nos. 1, 3 and 6 were converted into corresponding naphthenes by means of hydrogenation under atmospheric pressure and at 180—190° with the catalyst of Zelinsky (platinum on activated charcoal).

The properties of the hydrocarbons produced are as follows. 1, 3, 5-threemethyl -2-cyclopentyl -cyclohexane, b. p. 254—255° at 754 mm; $d_4^{20} = 0.8771$; $n_D^{20} = 1.4766$; $MR = 62.52$ (found); $MR = 62.45$ (calculated for $C_{14}H_{26}$). Maximum aniline point 59.0°.

Analysis:

0.1070 g subst.: 0.3398 g CO_2; 0.1282 g H_2O
0.1011 g subst.: 0.3205 g CO_2; 0.1225 g H_2O
Found %: C—86.60; 86.45; H—13.41; 13.56
Clcld. for $C_{14}H_{26}$%: C—86.51; H—13.49.

1-methyl-4-isopropylcyclopentylcyclohexane (cyclopentyl-para-methane), b. p. 271—273° at 752 mm; $d_4^{20} = 0.8765$; $n_D^{20} = 1.4771$; $MR = 67.14$ (found); $MR = 67.07$ (calculated for $C_{15}H_{28}$). Maximum aniline point 63.7°.

Analysis:

0.1120 g subst.: 0.3545 g CO_2; 0.1382 g H_2O
0.1074 g subst.: 0.3401 g CO_2; 0.1319 g H_2O
Found %: C—86.31; 86.36; H—13.81; 13.74
Clcld. for $C_{15}H_{28}$%: C—86.45; H—13.55.

Methylcyclohexylcyclohexane, b. p. 252—253° at 756 mm; $d_4^{20} = 0.8783$, $n_D^{20} = 1.4765$; $MR = 57.92$ (found); $MR = 57.83$ (calculated for $C_{13}H_{24}$). Maximum aniline point 55.5°.

Analysis:

0.1030 g subst.: 0.3267 g CO_2; 0.1246 g H_2O
0.1036 g subst.: 0.3290 g CO_2; 0.1246 g H_2O
Found %: C—86.50; 86.60; H—13.54; 13.47
Clcld. for $C_{13}H_{24}$%: C—86.58; H—13.42.

3-methyl-5-cyclopentylcyclohexane has previously been obtained by Petrov and Anzus [5]. By its properties it differs considerably from our hydrocarbon.

On the basis of data obtained it is possible to deduce some preliminary conclusions concerning the influence of side chains on the properties of aromatic hydrocarbons containing naphthenic nuclei. The addition of a methyl group to the hydrocarbons of the type of cyclopentyl-benzene pro-

duces but little effect on the values of specific weight and refractive index, but considerably increases its viscosity. By replacement of methyl groups by an isopropyl group all the properties are reduced. The influence of cyclopentyl rings in comparison with cyclohexyl nuclei is very striking; thus, mono- and dicyclopentyltoluenes have higher specific weights and refractive indices, but much lower viscosities than the corresponding cyclohexyltoluenes.

All the aromatic hydrocarbons have a high increment of molecular refraction; this is peculiar for polycyclic structures containing aromatic rings.

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BIOCHEMISTRY

MECHANISM OF WHEAT INJURY BY *EURYGASTER INTEGRICEPS*

By V. L. KRETOVICH, A. A. BUNDEL and K. V. PSHENOVA

(Communicated by A. N. Bach, Member of the Academy, 11. VIII. 1942)

Gluten of wheat kernels is destroyed by the attacks of wheat-bugs, because this grain contains highly active proteolytic enzymes. This has been proved by Kretovich and Tokareva ([1],[2]) who have been successful in isolating from injured grains dry enzymic preparations that decomposed gluten in a normal flour. In an earlier work ([3]) those authors have shown that the amylase is more active in injured grains than in normal ones.

It is natural to suppose that upon piercing a wheat kernel the wheat-bugs discharge into it their digestive juices which dissolve the contents of the grain, whereby the absorption of food by the bug is made easier.

In fact, Hibraoui ([4]) and Malenotti ([5]) have observed the wheat-bugs on pricking a grain of the vegetative organs of a plant to discharge a drop of a transparent liquid, which rapidly solidified in the air.

An attempt to discover proteinase and amylase in *Aelia cognata* has been made by Guillemet ([6]). By obtaining water extracts from entire insects or from separate parts of their bodies, he has shown that the body of the bugs contains amylase and proteinase; he has also found that extracts from heads were by far less active than those made from entire bodies. Naturally enough, such experiments when ground bugs are used for the extract are not fit to give an idea about the enzymes that are discharged by the bug when it attacks a grain or a leaf, for the highly differentiated structure of the bug is disturbed by the grinding. Therefore it is possible that the proteolytic and amylolytic activity observed by Guillemet was caused by the enzymes of the muscular tissue or the intestine rather than by the enzymes discharged by the bug from some excretory organ.

The source of the digestive juice discharged by wheat-bugs could evidently be discovered by means of a differentiated analysis of the enzymes of isolated parts of the alimentary canal. From this point of view great interest belongs to the salivary glands.

The present article contains the results of our investigation of *Eurygaster integriceps*, carried on in Kirghizia.

The salivary glands of *Eurygaster integriceps* are situated in the thoracic part of the bug, at either side of the oesophage: they can easily be extracted by carefully breaking off the head of the bug. Every salivary gland consists in a convoluted mass of very fine tubules fusing at their basis into a single excretory duct. The gland is from one to two millimeters across, its raw weight is about 1 mg. The anterior intestine, in which the food absorbed by the bug begins to be digested, fulfils here at once the function of the oesophage and stomach.

It is known that the wheat-bugs do not only suck out the contents of the ripening grain, thereby impairing their qualities as seed and technolo-

gical; they also cause a great harm by piercing the leaves and stalks of young plants, owing to which these lose vigour, become yellow and may even perish altogether.

In so far as a single generation of *Eurygaster integriceps* is hatched within a year, the same adult insects at various periods of their life feed on different parts of the plant; on grains at the ripening season, on leaves and stalks of young plants at the period between hibernation and egg-laying. It is therefore only natural to suppose that the enzymic apparatus of the digestive tube in *Eurygaster integriceps* is liable to change in the process of ontogeny according to the character of the food available. This is the reason why we investigated the enzymes of the digestive tube of *Eurygaster integriceps* twice, viz. at the period of stalk-formation and at the period of wax-ripening of the grain.

At the period of stalk-formation the salivary glands were found to be white, and the anterior intestine was tightly stuffed with a dark-green, nearly black material which had a disagreeable odour of digested herb. When ground in water, the salivary glands displayed a strong amylolytic action, which could be revealed by means of iodine-starch tests. Extracts made with the anterior intestine of wheat-bugs were also endowed with this property. Similar to the salivary glands, the anterior intestine as well was found to contain a proteinase able to split casein under the formation of products which defied precipitation with 2% CCl_3COOH.

Neither water nor sodium carbonate (0.25% Na_2CO_3) extracts from salivary glands of the anterior intestine were, however, found to impair the quality of the gluten of normal wheat flour.

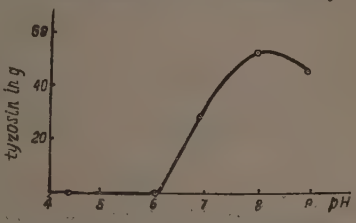
At the period of wax-ripeness of wheat the salivary glands were found to have a greenish-yellow hue, while the anterior intestine was considerably thicker and filled with some yellow-brown matter. The contents of the anterior intestine gave an exceptionally intense reaction of disamination, if treated with a mixture of acetic acid and nitrite, as well as with Millon's reagent. The reaction consisted in a profuse rising of nitrogen bubbles. This should de taken to mean that at the given stage of development of the bug, the content of the anterior intestine is very rich in products of enzymic splitting of protein. Extracts prepared from the salivary glands and the anterior intestine were found to contain active proteinase, that could be revealed after the accumulation of products unsettled with 2% CCl_3COOH, as well as a highly active amylase. Of great importance is the fact that sodium carbonate extracts prepared from salivary glands of a given collection were found totally to destroy the gluten of normal wheat flour and to lose their destructive properties upon boiling. Water extracts failed to destroy gluten until they were alkalized with sodium carbonate.

It is thus evident that at the period of grain ripening the salivary glands of *Eurygaster integriceps* contain, apart from amylase, a very active proteolytic enzyme. Hence it follows that the set of enzymes of the bug varies qualitatively, according to the stage of life of an adult individuum. As to the anterior intestine, aqueous extracts obtained from it have been found to produce an energetic destructive effect upon the gluten of normal flour; upon boiling, however, the proteolytic action of those extracts upon the gluten did not vanish altogether, but merely grew weaker. This circumstance pointed to the presence in the anterior intestine of substances stable against boiling and liable to activate the proteolytic enzymes contained in a normal flour. Activators of this kind, of the type of glutation, are known to contain sulphohydryl groups, and may be estimated quantitatively. The content of proteolysis activators in extracts prepared from anterior intestine has been measured by us according to Woodword and Fry's method as modified by Proskurjakov (7).

For titrating an extract prepared with 20 freshly isolated anterior intestines whose dry weight was 0.085 g, we took 2.0 mg of 0.001 *N* KIO_3.

According to Proskurjakov, the titration of an extract from 1 g of wheat germs, which are particularly rich in glutation, requires 4.6 mg of 0.001 N KIO_3. From a comparison of these data it becomes evident that at the period when the wheat-bugs feed on ripening grain, their anterior intestine contains an exceptionally large amount of sulphohydryl compounds. The digestion of the proteins absorbed in the anterior intestine of the bugs is facilitated by their reduction at the expense of sulphohydryl compounds; a similar process is known to occur in the alimentary canal of the larvae of garment moth.

What are then the properties of the proteinase discharged by the salivary glands of *Eurygaster integriceps*? The liquid discharged by the bug at the moment of stinging, upon treatment with a solution of bromine-thymol-blue, was found to have an alkaline reaction. It thus seemed reasonable to assume that the proteinase discharged by the salivary glands of the wheat-bug is a tryptase. In order to check up this assumption, we determined the activity of the proteinase of salivary glands at various pH with the help of Anson's method ([8]). As a substratum, use was made of Merk casein, respective values of pH being established by means of glycocoll-buffers. Analysis for «tyrosin» was made upon precipitation of the highly molecular nitrogeneous substances with 2% CCl_3COOH.



Proteinase of salivary glands of *Eurygaster integriceps* collected at the period of ripening.

The control experiments were made with a boiled enzymic extract. The results of experiments carried out with salivary glands at the period of grain ripening are shown in the figure. From this figure it is evident that the proteinase of the salivary glands of *Eurygaster integriceps* shows an optimum at a pH = 8.0, which makes it similar to the tryptase. That the proteinase of salivary glands is of the nature of tryptase is also evidenced by the data referred to above, viz. the destruction of the gluten of normal flour by sodium carbonate extracts from salivary glands, while aqueous extracts failed to produce the same effect. The latter circumstance is in full agreement with the high proteolytic activity of sodium carbonate extracts prepared from injured grains, as compared with water extracts ([2]).

In conclusion it should be noted that the data obtained do not only account for the mechanism of injuries caused to the grain by *Eurygaster integriceps*, but are also of interest for comparative physiology. They serve as an excellent illustration of the adaptability of the enzymes of the alimentary canal of an animal to food composition at various stages of its life.

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MINERALOGY

APATITES OF TWO TEXTURAL TYPES FROM APATITE-NEPHELINE ROCKS OF CHIBINY

By A. I. VOLODCHENKOVA and B. N. MELENTIEV

(Communicated by A. E. Fersman, Member of the Academy, 26.X.1942)

All the analyses of apatites from the tundra of Chibiny heretofore made have been made on samples collected according to morphological characters [1–3].

At this time, however, we may be interested in the analysis of such apatites as are typical of a given kind of rock, for the crystallization of certain rock-types is a function of the physical-chemical parameters formed in the present stage of the melt and characterizing the composition of the mineral. Yet, within a certain type of rock minerals of a single kind can possess different morphological characters. And at the same time, the quantitative relations of the minerals, belonging to morphologically different types, should be constant throughout the thickness of a particular rock and liable to no change but with the type of the rock. Therefore for the purposes of petrography and for the understanding of the chemical processes that were taking place during the crystallization of the rock, the composition of the average type of mineral is of major importance compared to the composition of an individual mineral of a particular outward appearance. So, we have set before ourselves the problem to establish the composition of the average apatite out of the different types of nepheline ores of Chibiny.

A specimen of such a kind may be separated from type-tests prepared on the basis of the textural features of the rock. For that purpose the apatite-nepheline ore was first subjected to centrifuging in neutral bromoform; then all of the magnetic minerals were removed from the heavy fraction by means of an electric magnet. The ultimate purification of the sample from concretions and other impurities was made under a binocular [4].

Specimen No. 161 was separated from a typical sample of mottled ores. The typical sample had been prepared by melting 722 furrow-samples. Another specimen, No. 165, was separated from a typical sample of breccia-like ores which had been composed of 110 furrow-samples.

Thus, there is good ground to assume that the separated apatite specimens represent the average kind of apatite, peculiar to the given textural type of ore.

Chemical analysis carried out by A. I. Volodchenkova has revealed nevertheless some admixtures of aegirine, fieldspars and nepheline. The results obtained have been corrected for the presence of nepheline in the specimen. In doing so all Al_2O_3 was referred to nepheline which made up 0.97% in the specimen No. 161, 0.55% in the specimen No. 165; the

remaining oxides (SiO_2, Na_2O and K_2O) were evaluated from the average analysis of nepheline. The results are referred to in Table 1.

Table 1

Content of Oxides Introduced with Nepheline

No. of specimens	Al_2O_3	SiO_2	Na_2O	K_2O
161	0.30	0.42	0.15	0.06
165	0.17	0.23	0.08	0.03

After nepheline and the residue insoluble in hydrochloric acid are eliminated from the gross-analysis, one obtains the data on pure apatites, which, referred to 100%, are presented in Table 2.

Table 2

Chemical Composition of Apatites

Components	No. 161	No. 165
P_2O_5	41.00	40.62
SiO_2	0.68	0.26
TR_2O_3	0.93	0.98
Fe_2O_3	0.05	0.29
CaO	52.99	52.85
SrO	2.02	2.57
MnO	0.11	0.07
MgO	traces	0.01
Na_2O	0.52	1.10
K_2O	0.19	0.21
F_2	2.60	1.79
	101.09	100.75
Minus O_2 equivalent to F_2	1.09	0.75
	100.00	100.00

Comparisons with the available analyses of apatites from the same deposits do not show any appreciable variations in the composition, except for the content of fluorine and alkalies.

Another interesting point to note is a rather close coincidence between the results obtained with sample No. 165 and those reported by Wlodavetz [3].

Thus, there is no need any longer to make different computations in order to ascertain the chemical composition of mottled and breccia-like rocks on the basis of an average analysis of apatite, since direct experimental data are always to be preferred to the results obtained by deduction.

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EXPERIMENTAL BIOLOGY

ON THE INFLUENCE OF PHYTONCIDES UPON *BACILLUS TUBERCULOSIS* OF THE STRAIN BCG

By G. NEBOLIUBOVA and B. TOKIN

(Communicated by L. A. Orbeli, Member of the Academy, 12. XI. 1942)

It has been [2,3] found that in the course of their life activity plants produce some substance of high bactericidal power, for which B. Tokin coined the name of phytoncides. After this discovery it seemed natural to test their effect upon a microorganism as resistant as *Bacillus tuberculosis*.

The present authors investigated the influence of the phytoncides of *Allium cepa* L. and *Allium sativum* L. upon the bacillus of the strain BCG. It is well known that Bertello, Calmette and Guérin [1] succeeded in obtaining this non-virulent strain not until 280 subsequent sowings were practised under identical conditions within 13 years.

With respect to its physical-chemical properties and resistivity the bacillus of this strain does not differ materially from that of the virulent strain; it preserves its original resistivity to acids and stains according to Ziel. Neither do the two strains essentially differ as to the content of fats, waxes, lipoids [2]. This is why any result obtained from experiments with bacilli of the strain BCG may safely be extended upon the virulent one, too. In our experiments we made use of the strain (Paris 588/d) obtained from the L. A. Tarassevich Institute in Moscow, and of the strain BCG from the Institute of Bacteriology and Epidemiology at Tomsk (this strain was subjected to passages at this laboratory).

Except a few modifications the technique adopted in our experiments was the following. The material to be re-sown into experimental vessels and test-tubes was kept for 5—8 min. in a box fixed to a platinum loop over phytoncide vapour evolved by a paste freshly prepared from onion or garlic. Control sowing was practised under usual conditions. It was impossible, however, to keep strictly sterile conditions without applying some supplementary synthetic antiseptics that might interfere with the accuracy of the experiment. But as the common flora is easily killed by the phytoncides of onion and garlic, it was unnecessary to take care of a special sterilization of paste, grounding utensils, etc.

As nutrient media use was made of thick or liquid potato media and of the medium of Sauton. After both experimental (phytoncide-treated) and control cultures had been re-sown, the glasses and dishes were transferred to a thermostat at 38°C and kept under daily observation.

When evaluating the results of our experiments, some technical details should be pointed out which made the success rather doubtful. First, the exposure was a short one (5—8 min.). Secondly, the bacilli to be treated were very abundant (several million), which was due to the technique of re-sowing. Thirdly, the phytoncides were to be tested as vapour. Fourthly,

when forming a relatively thick layer (which in the material to be re-sown may occasionally be as thick as 3 mm) the bacteria sometimes fail to be bombarded with phytoncides directly, especially those of them which adjoin the net of the platinum loop and are overlain by a layer of other bacilli. Lastly, it should be taken into consideration that the distance between the culture and the vegetable paste may occasionally amount to 1.5 cm.

And in spite of all these technical circumstances, the results obtained proved to exceed our expectation. Below is given a cumulative table of our experiments on phytoncides of *Allium cepa* L. and *Allium sativum* L.

Effect of Phytoncides of *Allium cepa* L. and *Allium sativum* L. upon *Bacillus tuberculosis* of the Strain BCG

No. of run	Number of exper. vessels		Number of control vessels	Specific growth						% of positive cases
				Exper. vessels				Identical for exper. and control vessels		
				Total lack		Inhibited				
	Phytoncides of onion	Phytoncides of garlic		onion	garlic	onion	garlic	onion	garlic	
1	11	4	4	7	2	3	2	1	—	93
2	3	3	2	2	1	1	2	—	—	100
3	9	—	3	—	—	—	—	9	—	0
4	3	3	2	—	—	—	—	3	3	0
5	9	—	2	5	—	3	—	1	—	00
6	15	—	3	15	—	—	—	—	—	100
7	5	7	3	1	7	4	—	—	—	189
8	12	—	4	5	—	5	—	2	—	83
9	6	—	2	6	—	—	—	—	—	100
Total	73	17	25	41	10	16	4	16	3	Average 80%
	90			71				19		

The experiments described above made us arrive at a conclusion that the tubercular bacillus of the strain BCG is killed after a five-minute treatment with phytoncides of *Allium cepa* L. or *Allium sativum* L.

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ENDOCRINOLOGY

ON THE QUESTION WHETHER THE GONADOTROPIC ACTIVITY OF THE PITUITARY BODY IN LOWER VERTEBRATES IS INCREASED BY Zn SALTS

By P. A. WUNDER

(Communicated by L. A. Orbeli, Member of the Academy, 3. X. 1942)

The ability of Zn salts to enhance the gonadotropic activity of extracts prepared with pituitary glands of mammals has been revealed by Maxwell (1) and supported by a number of authors. It was found soon, however, that the pituitary bodies of some mammals fail to show an increased gonadotropic activity when acted upon with Zn salts. To these belong the pituitary bodies of horse and man (2).

It has been found, on the other hand, that the increase produced in the gonadotropic effect by salts of Zn depends not only on the species of the mammal, whose pituitary body is under test, but on the species of the animal experimented upon, the recipient, as well. Thus, experiments on mice failed to give the phenomenon of an enhanced gonadotropic effect of hypophysal extracts (3).

This effect is also but poorly manifest in experiments with chicks (3). Consequently, the increase in the gonadotropic action of extracts from pituitary glands of mammals, as brought about by Zn salts, is a more complicated one than it appears to be at first sight. In this connexion it appeared of interest to investigate the way in which pituitary bodies of animals belonging to some other animal classes will respond to Zn salts, viz., to find out whether the effect of these salts will be to enhance the gonadotropic activity of their pituitary glands as well. The solution of this problem would certainly be interesting from the standpoint of comparative endocrinology and hormonal evolution.

Our experiments were carried out on amphibians, namely, on females of *Rana ridibunda*. In one of his earlier papers (4) the author has shown that by injecting intraperitoneally adult frog females with a suspension prepared from pituitary glands of males and females of *R. ridibunda*, at various periods of the year, one succeeds to call forth a premature ovulation, i. e. the discharge of eggs from the ovaries into the uterus. It has been found, moreover, that the effect is brought about with much ease in spring, in April, while in autumn and in winter it is rather difficult to attain. At these seasons it requires high doses of the pituitary gland suspension which is to be administered by small portions. Taking into account all that had been said above, the present experiments were run in two replications, in autumn 1941 and in spring 1942; as a positive reaction we considered the appearance of ovulation in *R. ridibunda* *.

* In some of these experiments the author was assisted by V. Yushenkova, undergraduate of this University.

I. Experiments on Autumnal Frogs. Fresh pituitary glands of *R. ridibunda* were ground in a mortar with 1/250 *n* $NaHCO_3$. The suspension obtained was divided into two equal portions, one of which was added with dry $ZnSO_4$ at a ratio of 1 mg per 1 cm^3 of the liquid. The suspension was injected subcutaneously, 0.5 cm^3 at once, by turns, into the left or the right thigh, twice a day. On the whole, two experiments were carried out; as a material, use was made of frogs in a different state, the dose of the pituitary gland being also varied as to experiment.

The frogs used in one of the experiments had been caught two weeks before. Such frogs usually show a weakened gonadotropic capacity of their pituitary glands and a reduced reactivity of their ovaries (4). The total dose of the pituitary gland was 1.8 g per frog; that of $ZnSO_4$, 1.9 mg per frog. In the other experiment all the frogs, both donors and recipients, were newly caught. The total dose of the pituitary gland was 3 glands per frog; that of $ZnSO_4$, 2 mg per frog.

The reaction was evaluated 24 hours after the last injection (48 hours after the beginning of the experiment) by means of dissection of the experimental frog. The degree of the reaction was estimated after the five-point system adopted by the author (4). The results are summarized in Table 1.

Table 1

Gonadotropic Activity of Pituitary Bodies of *R. ridibunda* as Affected by Zn Salts

Date of exper.	Dose of pituit. gland	Number of injections	Dose of $ZnSO_4$ in mg	Number of females recipients	Out of these reacted		Degree of reaction				
					absol.	in %	±	+	++	+++	++++
1941 30.IX—1.X	1.8 of a gland, mostly from males*	4 (2×2)	1.9	5*	3	60	1	—	1	1	—
Do	1.8 of a gland*	4	—	5*	—	—	—	—	—	—	—
14—15.X	3 glands from females or males	4	2	10	7	70	2	1	1	2	1
Do	Do	4	—	10	0	0	—	—	—	—	—

As may be seen from Table 1, a subcutaneous injection of soda suspension from pituitary glands, made at a dose of 1.8 and 3 glands per frog, failed to produce any reaction at all, and no ovulation set in. Reaction was brought about by the same hypophysal suspension injected at the same dose, but with an addition of $ZnSO_4$. In an experiment with a dose as high as 3 pituitary glands a reaction was obtained in 70 cases out of 100; in 4 cases the rate of ovulation was a rather high one (from ++ to ++++). Even in an experiment made with a small dose (1.8 of a pituitary gland), on frogs kept in a vivarium during 2 weeks (which certainly impaired their reactive capacity), a gonadotropic reaction set in. Hence the conclusion should be made that the effect of Zn salts added to a suspension prepared

* All the frogs, both donors and recipients, were kept in a vivarium two weeks prior to experiment.

from pituitary glands of *R. ridibunda* is sharply to enhance the action of the suspension on females of the same species.

II. Experiments on Spring Frogs. The second run of experiments was carried on according to a different method. Allowing for the fact that in spring females of *R. ridibunda* are more susceptible to the treatment with hypophysal suspension than they are during autumn, we considerably reduced the dose, making it as low as 0.5 of a gland per female, instead of the 3 glands administered in the first experimental run. Besides, the dose was injected at once, intramuscularly. Otherwise the procedure remained without changes. The suspension prepared with 1/250 *n* $NaHCO_3$ was divided into two portions: one of the portions was added $ZnSO_4$ at a ratio of 1 mg per 1 cm^3 of the suspension. The total dose of the pituitary gland per one female was thus 0.5 of a male-gland. The dose of $ZnSO_4$ was 0.5 mg. The results are given in Table 2.

Table 2

Gonadotropic Activity of Pituitary Bodies of *R. ridibunda* as Affected by Zn Salts

Date	Dose of pituit. gland	Number of injections	Dose of $ZnSO_4$ in mg	Number of females recipients	Out of these reacted		Degree of reaction				
					absol.	in %	±	+	++	+++	++++
1942 30.IV	0.5 of male gland	1	0.5	7	6	85.7	—	2	1	3	1
Do	Do	1	—	9	2	22.2	1	—	—	—	1

From an analysis of Table 2 we become aware of on enhancing effect of the Zn salts, which is manifest even in the experiment made with a very small dose of the pituitary gland. When added to a soda suspension prepared with pituitary glands of *R. ridibunda*, salts of Zn sharply increase the effect of this suspension, which is to bring about a premature discharge of the eggs from the ovary. As seen from the experiments, an addition of Zn salts to the hypophysal suspension does not tell upon the quality of the eggs produced in the process of artificially provoked ovulation. When artificially fertilized, the eggs thus obtained showed a normal course of development.

Summing up, it should be noted that salts of Zn are apt to enhance the hormonal activity of the pituitary body not in mammals alone, but in lower vertebrates (amphibians) as well.

The data obtained by the author make it seem very likely that salts of Zn may increase the gonadotropic effect produced by the pituitary glands not only of amphibians, but also of fish. The latter observation may be applied on a commercial scale.

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